Rounding Again

A notable feature of the above calculation is the systematic (though implicit) way in which insignificant quantities are discarded or "zeroed", through rounding. The "general" rule for rounding was rounding to the nearest integer, so that a quantity greater than $\frac{1}{2}$ was rounded up to the next higher figure. But we have already seen an exception to this rule. In fact, as in Pāṇinī's grammar (or in the way traffic rules are still observed in smaller towns in India today) there are a very large number of exceptions! We find in Ranganatha[43] a comment about how to round the 24 sine values in exceptional cases which call for a departure from the rule.

> In the 21st, 20th, 6th, 15th, 7th, 12th, & 17th there is a difference ....

That is about 30% cases in which there is an exception!

In any case, it is clear that there was no *mechanical* rule in use for rounding, and that rounding as appropriate to ultimately greater precision was used. Thus, the numbers used in Indian mathematics, though very similar to floating point numbers, did not correspond *exactly* to any specific type of floating point numbers actually being used today, all of which involve mechanical rules for rounding.

There is a fundamental philosophical difference here, for it seems unlikely (and I believe it to be impossible) that one can at all reduce a purposive procedure to a mechanical (or causal) rule needed for routine numerical computing on a digital computer. To bring out this subtle philosophical difference between a purposive procedure, and a mechanical one, one can ask the question: would it be possible to design an expert system or an artificially intelligent computer which could mechanically reproduce such a purposive approach? This question is interesting because, as we have seen, Hilbert's vision of mathematics is so profoundly mechanical. This is too big a question to discuss here; however, I can summarize an answer that I have provided elsewhere:[44] a truly purposive procedure cannot, in principle, be mechanized. Thus, though Indian mathematics was computational, given these scarcely noticeable features, it may well be that there is a very fundamental philosophical difference between computation in Indian mathematics, and present-day rule-based computational mathematics.

The subtle difference may perhaps be more easily explained, in a non-technical way, by means of an analogy, readily comprehensible to those familiar with the difference between Indian and Western music. In Western music, the phenomenon known as the "Pythagorean comma" creates a problem analogous to the problem of rounding: starting from a given note, if one ascends 12 times by perfect fifths, then this is not the same as ascending by 7 octaves. (Alternatively, if one builds a scale of 12 notes by raising each note to a perfect fifth, and then reducing these 12 notes to the primary octave, then the 12th note in this scale will not be a perfect octave of the base note, so that these twelve notes will not form a perfect cycle—the musical cosmos fails to be exactly recurrent!) The failure of the musical cosmos to be recurrent is a catastrophe from the Pythagorean viewpoint. Since the perfect fifth (on the Pythagorean scale) is understood to have a frequency in the ratio of 3/2 to the frequency of the base note, and since an octave has double the frequency of the base note, the difference amounts to the ratio $\frac{(3/2)^{12}}{(2/1)^7} = \frac{531441}{524288} \approx 1.0136432$, which differs very slightly from 1. However, Western music presupposes that 12 perfect fifths are *exactly* equal to 7 octaves. No easy *mechanical* rule is available to settle the problem of the "Pythagorean comma". However, in the West, the common instruments for music, like the piano, are mechanical, in the sense that they are given, and not open to tuning by the player. Therefore, a mechanical rule was thought desirable. Hence, the actual solution, that is today in use, is called the equal-tempered scale, which flattens each note by a small amount. (The notes on the equal tempered scale are obtained by ascending by $\sqrt[12]{2}$, so that the 12 notes fit into a perfect octave.) While this solves the problem of the Pythagorean comma, and also standardizes all instruments, it also has the disadvantage that it makes *every* note in Western music very slightly off key. Though scarcely noticeable except to a musically trained ear, this is a very unsatisfying consequence of marrying a mechanistic philosophy to something like music which seems intrinsically non-mechanical. With traditional Indian musical instruments, however, even a "fixed-pitch" instrument like a flute is so designed as to admit of substantial human adjustment during play. (Also, there is no compulsion to follow a pre-prepared musical score, which might have been composed by another person, using a different instrument.) Tonal problems, therefore, are left to be resolved by the expert player in real time without the need to degrade, even if ever so slightly, the quality of the music as a whole.

Finite Differences vs Square Roots

To return to the calculation of trigonometric values, it is evident that the numerical method is shockingly easy compared to the geometrical method using triangles and square-root extraction. Using the stored table of differences, which are themselves small numbers, only simple addition is required. Even if the differences themselves are to be computed, the multiplication in (3.28) involves relatively small numbers, and absolutely no square-root extraction is necessary. (Somehow this point seems to have been overlooked by more recent commentators on Āryabhaṭa.) These differences become all the more important when we take into account the rounding that must necessarily accompany actual square-root extraction in the geometrical method.

The geometric method, apart from being quite cumbersome when a large number of sine values are involved, especially when square roots have actually to be extracted (and not merely indicated symbolically), has a further disadvantage: the geometric method enables the computation of sine values only at a discrete set of points. This is the third key point to observe regarding the *Gaṇita* 12 rule: it facilitates interpolation.